TRAVERSING THE EPISTEMOLOGY OF PROBABILITY IN INDIAN MATHEMATICS TEXTBOOKS

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The purpose of this paper was to study the views adopted by Indian curriculum makers, regarding the nature of probability, as reflected through textbooks, and to reflect on the possibilities and challenges of including the epistemologies of probabilities in the school curriculum. To understand the notions and practices that have been associated with the epistemology of probability, NCERT Mathematics textbooks (written after The Kothari Commission, 1966) were examined, reviewed and analyzed in a historically-chronological manner. As predicted, the classical and the experimental interpretations of probability dominants the school mathematics curriculum and the subjective approach remains uncovered in school textbooks, even today.

INTRODUCTION

Every subject domain espouses several knowledges, but to understand the kind(s) of knowledge children gather, tacitly, a prima facie instrument to study is the textbooks. The decision of content selection, in-turn of knowledge, lies with curriculum makers and on the choice of textbook writers. Textbooks, thus, become agencies through which knowledge is transformed and constructed (Chevallard, 1988; Kang & Kilpatrick 1992, call this didactic transposition), and through which the envisioned goals and objectives of a curriculum are concretised (Ball & Cohen, 1996). Textbooks, particularly in mathematics, act as mediators through which the dispositions and intentions of curriculum framers, textbook writers and mathematicians are dispensed to the students.

Thus, if one needs to know how and what kind of understanding students form about the nature of a domain, Mesa (2004) emphasises on analyzing textbooks, stating that textbook-analysis could help in answering several questions (though hypothetical) related to the linkages between the intended curriculum envisaged by curriculum or policy makers and the attained curriculum that is actually learned by the students. Analyzing content helps one discover the authors' epistemological decisions while selecting a specific content -what has led to the legitimization of a preferred piece of knowledge (Apple, 1986); what would students learn, if they followed the text entirely (Mesa, 2004); how would students construct meaning from the text (Weinberg & Wiesner, 2011); and what notions would the students develop by reading the text alone (Kang & Kilpatrick, 1992).

This paper has been conceptualized to study the epistemologies adopted in Indian mathematics textbooks specific to the domain of probability.

The nature of defining and interpreting probability, as a mathematical construct, has undergone significant epistemological shifts. Since textbooks are the first, and sometimes, the only source of knowledge transmission, I was keen to study if our textbooks have kept pace with this evolving nature of probability. An attempt has been made to study which approaches

of interpreting probability have found a place in the mathematics curriculum, and, further, to discuss their implications in promoting probabilistic thinking.

UNDERSTANDING PROBABLITY

As a broader framework, the two main approaches to define and understand probability include the Objective Approach and the Subjective Approach. The Objective Approach can be, further, subdivided into the Classical and Frequentists approach (Batanero, Henry & Parzysz, 2010; Konold, 1989; Prodromou, 2012).

Classical Approach of Defining Probability

For classicists (also referred to as theorists), probability emerged from the idea of proportionality and they state that the probability of happening or not happening of a 'favourable' event is dependent on a combination of all the outcomes that should be equi-probable. This notion is quite analogous to the part-whole relationship of a fraction. Consequently, to find the probability of an event, one needs to enumerate all the possible combinations and, then, consider the proportion between the desired event and all the enumerated possibilities. An inherent assumption of the classical approach is on equal likeliness of the events. Thus, in the classical approach, probability of an event is considered a priori.

Frequentists' View to Quantify Probability

The proponents of the frequentist (empiricist or experimental) probability regard the probability of a simple event through observations of the trend of relative frequencies, obtained from repeated trials. Based on Bernoulli's principle of the law of large numbers, the probability of an event is empirically (rather hypothetically) based on the principle of stabilization of frequencies, after a repetition of similar trials. Thus, for the frequentists, quantification of probability is embedded in the physical properties of the object or the randomiser. Therefore, since this observable randomness can be produced or stimulated, assigning frequentist probability to a single case event is, often, not easy.

From a pedagogical viewpoint, Fischbein (1975) elaborates experimental probability learning as being associated with: a) a specific experimental paradigm and b) a probability matching paradigm. Within the specific experimental paradigm, a person is presented with a succession of trials and, on each trial; one is required to predict the outcome before it occurs. For example, predict the chances of getting a black or a white ball before the draw is made. In the probability matching paradigm, a sequence (unknown to the participant), randomly determined by some process, with the probability of each outcome being fixed, is presented and the person is expected to approximate the probabilities of the respective outcome based on its relative frequencies. The core idea is to maximise the probability of success on every trial, hence, it is also termed as expected probability.

Subjectivists' Interpretation of Probability

The conception of probability being subjective emerged from Bayes' assumptions that considered probability of an event being based on observable consequences. According to the subjectivists, probability is an expression of personal beliefs or of perceptions related to an event. Since probability is an estimated value of a future event, its value will depend upon numerous factors, such as the observer's knowledge, conditions under which the observations were made and the models that were used to represent the situation (Batanero, Henry &

Parzysz, 2010). Probability, according to the subjectivists, cannot be deterministic. Rather, it would be subject to revisions, based on the availability and re-availability of information. Subjective probability opens opportunities to different descriptions of the same event and, thus, to different judgments. As Kyburg, 1974 (cited in Batanero et al, 2010, p. 24) explains, "randomness [for subjectivists], is no longer a physical objective property, but has a subjective character and probability does not measure a magnitude, such as length or weight, but only a degree of uncertainty, specific to each person".

A Comparison of the Three Interpretations

The main controversy between the subjectivists' viewpoint and that of the others, is in the embeddedness of the idea. According to the objectivists, probability can be assigned to an event that can be repeated, while for the subjectivists, probability is interpreted in the degree of belief, meaning, thereby, that the probability of an event is entrenched in the mind, rather than in the object that created randomness. For the subjectivists, probabilistic judgments are manifestations of personal judgments (Hawkins & Kapadia, 1984). For the frequentists, however, probability is embedded in the objects of randomness. There is, thus, an inherent reliance on the physical, mainly on symmetrical properties of the randomisers. Subjectivists can assign probability to a single, individual event, but, since frequentists calculate probability, as a limit towards which relative frequencies of an infinite class of similar events tend, their presumption rests on equal-likeliness of a class of events.

METHOD

This work has been organised by a series of broad questions: Which approaches of understanding probability have been espoused in the textbooks?, How have the NCERT textbooks presented the content of probability?, Have the textbooks done justice in representing the three views of probability?, Which approach(es) has(ve) been most dominant with curriculum makers?.

Taking off from where the textbooks emerged as a teaching resource in India, i.e. NCERT textbooks written after the first policy on school education of independent India, (The Kothari Commission, 1966), school mathematics textbooks published by NCERT were studied for content on probability. In order to know how, if at all, the notions of understanding probability have changed in these 50 years, the books were traversed in a historically-chronological way. Thus, I started to review the mathematics textbooks written after 1966 to the most recent ones.

In these, almost 50 years, there have been five phases of development and revision of textbooks: 1966-1975, 1975-1986, 1987-2000, 2000-2005 and 2005-current year. From each phase, one textbook, from each grade, was selected as a representative sample.

To understand the perspectives of probability, textbooks from all the five phases were studied on any chapter/section that was titled 'Probability', 'Chance' or 'Data'. In all such selected chapters/sections, all tasks (problems, exercises, text that preceded the exercises, including examples) were considered. For each task, the following four questions were answered: What is the main idea of the task? In order to attempt the task, what does the author expect the reader to do? Does the task reflect any particular approach? In solving the task, which approach is the most dominant? The responses to these questions helped in the conclusion of the larger question: Which notions of probability are most preferred by the curriculum framers or textbook writers.

During the analysis, it was found that the responses on a task, obtained from any of the four questions given above, overlapped with each other. Thus, the questions complemented each other in intent.

ANALYSIS

The first, so-called, textbooks in mathematics for the school level in India were produced under a detailed programme undertaken by the Central Committee on Education, set up by The National Council, in 1961. Subsequently, the textbooks were released in 1969. In this first phase (1969-75) of textbook development, the objective of teaching mathematics at the senior secondary classes (considered, then as Class X and Class XI) was to help students to acquire an understanding of the nature of mathematical definitions and of principles of mathematical proofs (foreword NCERT, 1969). In these textbooks, I was unable to find any chapter on either statistics or probability. Thus, one can conclude that in those years (1969-75), probability and statistics were not taught to school students.

Probability emerged in the senior secondary textbooks only in 1975. From 1975-86, NCERT had launched a combined series of five books for Classes XI and XII. Topics related to calculus, coordinate geometry, complex numbers and higher algebra were introduced as part of high school mathematics curriculum. In Part IV of this series, entitled 'Higher Algebra', there were two chapters on probability: ‘Chapter XXVII: Chance in Mathematics: An Introduction to Probability’ and ‘Chapter XXVIII: Random Variable and Probability distribution’. As is evident from their titles, the former chapter treated probability in an axiomatic way and the latter chapter provided an experimental approach of calculating probabilities. In the first chapter (Chapter XXVII, NCERT 1978), probability had been considered as a measure of chance, but the chapter did not provide any context or explanation of what chance and unpredictability are. The reader does not get to know the real meaning of fortuitousness. Big ideas, such as of mutually exclusive events, independent events and addition rules, were mentioned, but treated as priori constructs. The content was dominated by Kolmogorov’s axioms of probability. In the second chapter, (Chapter XXVIII, NCERT 1978) objects that can be used to generate randomness, such as coins, dies, playing cards and so on, have been extensively referred to. Ideas of discrete random variables and probability distributions were given in this chapter. Most of the tasks were based on the idea of pay offs or of winning that get established via experiments with games of chance. The concept of probability distribution, as patterns emerging from repeated experiments with devices of randomness (dice, cards, coins), find a mention. This chapter, though projected in the frequentists’ paradigm, fails to meet to its true nature. Examples and exercises reflect a static, dull vision, devoid of any actual stimulation or experimentation. By dividing the content in two separate chapters, it was evident that the text book writers had perceived the axiomatic nature and the experimental nature as being distinct and no cord could be established between the two approaches.

In the textbooks written in 1986-2000, the content of probability was squeezed in a single chapter of Class XII textbook. In its 15 sub-sections, the chapter covers all the major ideas, such as of random events, sample space, theorems of probability, conditional probability, random variables, probability distributions and binomial distribution. On analyzing the text, it was found that this chapter, too, professed a very theoretical, definitive and an algebraic way of dealing with probability.

Following the recommendations of the National Curriculum Framework for School Education-2000, the mathematics textbooks were revised again and adopted from 2000-2004. In this temporal phase, the curriculum makers felt that since, at the higher secondary stages (Classes XI and XII) mathematics is pursued as a subject of choice, and since the expectations from mathematics for science students would be different from those who take commerce, the mathematics course at higher secondary classes must be divided into compulsory and elective components. Thus, the textbooks for these grades were divided into three parts, A, B and C. The 'A+B' combination was offered to science students and the 'A+C' combination was offered to the commerce stream students. A chapter on Probability was included in the compulsory component i.e. Part-A of the Class XII (NCERT, 2003) textbook. In this 45 page chapter, topics related to random experiments and sample space, probability of an event, theorems of probability, random distributions and applications were mentioned. Akin to its predecessors, here, again, most of the content followed an axiomatic approach, with sporadic instances of use of experimentation and randomisers. In addition, many examples, problems and exercises were similar to those of yesteryears. The content of this phase was rather dull, disappointing and mostly repetitive.

In the mathematics textbooks, written after the National Curriculum Framework, 2005, probability, for the first time, finds a place in the middle school curriculum (Class VII, onwards). In the Class VII mathematics textbooks, a section titled 'Chance and Probability', in the chapter, 'Data Handling', hints of a dynamic outlook towards doing probability. Readers are expected to first make predictions, then conduct experiments with random generators and, finally, match the closeness of their prediction to their observations. The idea of quantifying prediction, as a pre-construct of an experiment, is established, thus introducing the learners to the idea of quantifying fortuitousness. "If you toss a coin, can you always correctly predict what you will get? Try tossing a coin and predicting the outcome each time…You will find that the observations show no clear pattern...it is a matter of chance that in a particular throw you get either of these [Head or Tail]", (NCERT 2006a pg. 74). The chapter does not establish any a priori approach to probabilistic experiments and brings out the essence of doing probability, as based on unpredicted patterns. In Class VIII textbooks, the section on probability begins with a subjective approach, but soon turns to the old school of objectivism. Some of the examples have gone too far in assuming equal-likeliness of events. In one such task, weather predictions are being made based on the trend of yesteryears. Equal-likeliness has, thus, been assumed even on weather: "What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then 1/10. The probability that it does not rain = 9/10 … The use of probability is made in various cases in real life... Metrological department predicts the weather by observing trends from the data over the past many years", (NCERT 2006b, pgs. 86-87). From Class IX onwards, probability enjoys the status of a separate, independent chapter. In the Class IX textbook, probability has been expressed as a measure, a ratio of frequencies (the authors call it a "statistical approach of probability"). In Class X, the theoretical approach emerges as a dominating feature and this continues to the higher classes. In Class XII, the idea of conditional probability, based on calculating the probability of occurrence of an event, followed by the occurrence of an already occurred event, has been presented, but the examples draw over non-real, artificial contexts. Here's an example: "A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3 and 1/12, if he comes by train, bus and scoter

respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?" (NCERT 2006f, p. 554).

RESULTS

While analyzing the textbooks, it was evident that probability has been a neglected area in school mathematics for long. Such a conjecture can be made over two observations: first, a quick glance on the Tables of Content reveals that, in all these years, except after 2005, probability was mostly presented as the last chapter of the book. If sequencing of chapters has any significance, even subtly, in establishing the worthiness of a domain, then this feature is worth noting; second, till 2005, probability was taught only in the higher grades of schooling. One of the possible reasons for neglecting probability in junior classes can be attributed to the Piagetian influence on deciding the maturity level of children to acquire a particular concept. For long, developmental theories have restricted several mathematical concepts to be taught below a certain maturity level. It was felt that younger children have no notion of chance and, thus, probability should be introduced only after the onset of the formal operational stage, that is, in higher grades (Piaget & Inhelder, 1975). Though one is not sure if the Indian mathematics curriculum framers were also influenced by these theories, the absence of even the simplistic notions related to chance in the junior level textbooks, written before 2005, lead to such an inference. It is only after 2005, with the new vision of a school mathematics curriculum, that concepts related to probability are now being introduced in the middle school.

It has been noted, across textbooks, that the content of probability has remained dominated by classical interpretations, and only in very few instances has the experimental notion been introduced. Most of the concepts were stated in terms of a definitive relation between the total events and the ones to occur: "the number of desired outcomes divided by the number of possible outcomes." Only about 10% of the problems were stated in the experimental (frequency) approach.

Further, till 2005, there existed a stark divide between classical and experimental approaches. So much so, that in the textbooks of 1975-2000 these approaches have been treated in separate, independent sub-sections, if not as separate chapters. No linkages could be traced between the two aspects. It is only after 2005 that an attempt has been made, at least in the middle grade level, only at introductory stages, to link classicists and experimentalists.

REFLECTIONS AND SUGGESTIONS

Across the years, it has been found that the content of probability has been dominated by the axiomatic (theoretical) approach, as espoused by Kolmogorov, in the 20^{th} century. The Axiomatic approach, though convenient for determining probability, is fraught with some limitations. To begin with, Kolmogorov's work, as one would expect, is very formal, precise and relatively complicated and hard to understand by non-mathematicians. The Axiomatic approach gives no clue to the real meaning of probability. Since, at the school level, students ought to be exposed to the fundamental meanings, being confined to only the theoretical aspects will present a rather narrow, insufficient way of looking at probability.

Simultaneously, it is imperative to comment on the fact that only talking about the experimental approach will also delimit the essence of probability. Probability is a study of fortuitousness of an event wherein we quantifying the chances of an event yet to happen. Contrary to this basic idea, empiricists, rather calculate probability on the premise of

replicating events, on the assumption of an already existing pattern and then determine its uncertainty. In order to imbibe the experimental approach in textbooks, appropriate pedagogy has to be followed, such as making predictions, conducting experiments and, finally, matching the closeness of prediction to the outcomes.

Often, probability is neither objective nor open to a frequency interpretation. The third approach, i.e. the subjective approach, which counts on intuitive conceptions has not found its due place in the textbooks. It must be acknowledged that the first conception that many children form of any uncertain situation, comes from their informal subjective experiences. When teaching probability, some account should be made of informal, nascent perceptions that children hold regarding chance and uncertainty. Since subjective probability relies on a comparison of perceived likelihood, it can be used to induct not -so -mathematically-sophisticated children or primary children to the idea of guessing fortuitousness. Moreover, to teach objective probability, some acquaintance with fractions is required, but children who are not-so-good with fractions or those at the earlier stages of schooling can be helped by encouraging them to present their intuitive or subjective views about chance and probability. The subjective ideas can, thus, be encouraged at the primary level. They can even be taken as precursors, before quantifying, to help initiate meaningful communication in an otherwise dull, calculation dominated probability class (Hawkins & Kapadia, 1984; Borovcnik & Kapadia, 2009).

Thus, to encourage probabilistic thinking, the content should be presented in a blended mode, deriving from the strengths of each interpretation of probability. Teaching probability at the school level should present enough intellectual excitement and scope for students to express their intuitions (subjectivity), weigh evidences (empirically), make meaningful quantifications based on strong arguments and, finally, establish a relationship between their judgments and computation.

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